# Mathematica 11.3 Integration Test Results

## Test results for the 298 problems in "1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q.m"

### Problem 220: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2}\left(A+B x^2\right)\sqrt{b x^2+c x^4}\,dx$$

Optimal (type 4, 243 leaves, 7 steps):

$$\frac{4 b^2\left(3 b B-5 A c\right)\sqrt{b x^2+c x^4}}{231 c^3\sqrt{x}}-\frac{4 b\left(3 b B-5 A c\right)x^{3/2}\sqrt{b x^2+c x^4}}{385 c^2}-\frac{2\left(3 b B-5 A c\right)x^{7/2}\sqrt{b x^2+c x^4}}{55 c}+\frac{2 B x^{3/2}\left(b x^2+c x^4\right)^{3/2}}{15 c}-\left(2 b^{11/4}\left(3 b B-5 A c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(231 c^{13/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 177 leaves):

$$\frac{1}{1155 c^3}2\sqrt{x^2\left(b+c x^2\right)}\left(\frac{1}{\sqrt{x}}\left(30 b^3 B+2 b c^2 x^2\left(15 A+7 B x^2\right)-2 b^2 c\left(25 A+9 B x^2\right)+7 c^3 x^4\left(15 A+11 B x^2\right)\right)+\left(10\,\mathrm{i}\,b^3\left(-3 b B+5 A c\right)\sqrt{1+\frac{b}{c x^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)\right)\right)$$

## Problem 221: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2}\left(A+B x^2\right)\sqrt{b x^2+c x^4}\,dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$\frac{4 b^2 (7 b B-13 A c) x^{3/2}\left(b+c x^2\right)}{195 c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b x^2+c x^4}}-\frac{4 b (7 b B-13 A c)\sqrt{x}\sqrt{b x^2+c x^4}}{585 c^2}-\frac{2 (7 b B-13 A c) x^{5/2}\sqrt{b x^2+c x^4}}{117 c}+\frac{2 B\sqrt{x}\left(b x^2+c x^4\right)^{3/2}}{13 c}-\left(4 b^{9/4}(7 b B-13 A c)\, x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(195 c^{11/4}\sqrt{b x^2+c x^4}\right)+\left(2 b^{9/4}(7 b B-13 A c)\, x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(195 c^{11/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 273 leaves):

$$\frac{1}{195 x}\sqrt{x^2\left(b+c x^2\right)}\left(\frac{2 x^{3/2}\left(-14 b^2 B+2 b c\left(13 A+5 B x^2\right)+5 c^2 x^2\left(13 A+9 B x^2\right)\right)}{3 c^2}+\left(4 b^2 (7 b B-13 A c)\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)-\sqrt{b}\sqrt{c}\sqrt{1+\frac{b}{c x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\sqrt{b}\sqrt{c}\sqrt{1+\frac{b}{c x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,c^3\sqrt{x}\left(b+c x^2\right)\right)\right)$$

## Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{x}\left(A+B x^2\right)\sqrt{b x^2+c x^4}\,dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$-\frac{4 b (5 b B-11 A c) \sqrt{b x^2+c x^4}}{231 c^2 \sqrt{x}}-\frac{2 (5 b B-11 A c) x^{3/2} \sqrt{b x^2+c x^4}}{77 c}+\frac{2 B \left(b x^2+c x^4\right)^{3/2}}{11 c \sqrt{x}}+\left(2 b^{7/4} (5 b B-11 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(231 c^{9/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 159 leaves):

$$\frac{1}{231} \sqrt{x^2 \left(b+c x^2\right)} \left(\frac{-20 b^2 B+4 b c \left(11 A+3 B x^2\right)+6 c^2 x^2 \left(11 A+7 B x^2\right)}{c^2 \sqrt{x}}+\left(4 i b^2 (5 b B-11 A c) \sqrt{1+\frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 \left(b+c x^2\right)\right)\right)$$

## Problem 223: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \sqrt{b x^2+c x^4}}{\sqrt{x}} \, dx$$

Optimal (type 4, 326 leaves, 7 steps):

$$-\frac{4 b (b B-3 A c) x^{3/2} \left(b+c x^2\right)}{15 c^{3/2} \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{b x^2+c x^4}}-\frac{2 (b B-3 A c) \sqrt{x} \sqrt{b x^2+c x^4}}{15 c}+\frac{2 B \left(b x^2+c x^4\right)^{3/2}}{9 c x^{3/2}}+\left(4 b^{5/4} (b B-3 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(15 c^{7/4} \sqrt{b x^2+c x^4}\right)-\left(2 b^{5/4} (b B-3 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(15 c^{7/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 247 leaves):

$$\frac{1}{15\,x}\sqrt{x^2\left(b+c\,x^2\right)}\left(\frac{2\,x^{3/2}\left(2\,b\,B+9\,A\,c+5\,B\,c\,x^2\right)}{3\,c}-\left(4\,b\left(b\,B-3\,A\,c\right)\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c\,x^2\right)-\sqrt{b}\,\sqrt{c}\,\sqrt{1+\frac{b}{c\,x^2}}\;x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\sqrt{b}\,\sqrt{c}\,\sqrt{1+\frac{b}{c\,x^2}}\;x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\;c^2\sqrt{x}\left(b+c\,x^2\right)\right)\right)$$

## Problem 224: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\sqrt{b\,x^2+c\,x^4}}{x^{3/2}}\,dx$$

Optimal (type 4, 165 leaves, 5 steps):

$$-\frac{2\left(b\,B-7\,A\,c\right)\sqrt{b\,x^2+c\,x^4}}{21\,c\,\sqrt{x}}+\frac{2\,B\left(b\,x^2+c\,x^4\right)^{3/2}}{7\,c\,x^{5/2}}-\left(2\,b^{3/4}\left(b\,B-7\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(21\,c^{5/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 134 leaves):

$$\frac{1}{21}\sqrt{x^2\left(b+c\,x^2\right)}\left(\frac{2\left(2\,b\,B+7\,A\,c+3\,B\,c\,x^2\right)}{c\,\sqrt{x}}-\frac{4\,i\,b\left(b\,B-7\,A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\;c\left(b+c\,x^2\right)}\right)$$

## Problem 225: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \sqrt{b x^2+c x^4}}{x^{5/2}} \, dx$$

Optimal (type 4, 323 leaves, 7 steps):

$$\frac{4 \left(b B+5 A c\right) x^{3/2} \left(b+c x^2\right)}{5 \sqrt{c} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}+\frac{2 \left(b B+5 A c\right) \sqrt{x} \sqrt{b x^2+c x^4}}{5 b}-\frac{2 A \left(b x^2+c x^4\right)^{3/2}}{b x^{7/2}}-$$
$$\left(4 b^{1/4} \left(b B+5 A c\right) x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(5 c^{3/4} \sqrt{b x^2+c x^4}\right)+$$
$$\left(2 b^{1/4} \left(b B+5 A c\right) x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(5 c^{3/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 218 leaves):

$$\frac{1}{5 x} \sqrt{x^2 \left(b+c x^2\right)} \left(\frac{2 \left(2 b B+5 A c+B c x^2\right)}{c \sqrt{x}}+\frac{1}{b+c x^2}\right.$$
$$4 i \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \left(b B+5 A c\right) \sqrt{1+\frac{b}{c x^2}}\, x\, \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]-$$
$$\left.\frac{1}{b+c x^2} 4 i \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \left(b B+5 A c\right) \sqrt{1+\frac{b}{c x^2}}\, x\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)$$

## Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \sqrt{b x^2+c x^4}}{x^{7/2}} \, dx$$

Optimal (type 4, 163 leaves, 5 steps):

$$\frac{2\left(b B+A c\right)\sqrt{b x^2+c x^4}}{3 b\sqrt{x}}-\frac{2 A\left(b x^2+c x^4\right)^{3/2}}{3 b x^{9/2}}+$$
$$\left(2\left(b B+A c\right) x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(3 b^{1/4} c^{1/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 119 leaves):

$$\frac{1}{3}\sqrt{x^2\left(b+c x^2\right)}\left(\frac{2\left(-A+B x^2\right)}{x^{5/2}}+\frac{4\,\mathrm{i}\left(b B+A c\right)\sqrt{1+\frac{b}{c x^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)}\right)$$

## Problem 227: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B x^2\right)\sqrt{b x^2+c x^4}}{x^{9/2}}\,dx$$

Optimal (type 4, 328 leaves, 7 steps):

$$\frac{4\sqrt{c}\left(5 b B+A c\right) x^{3/2}\left(b+c x^2\right)}{5 b\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{b x^2+c x^4}}-\frac{2\left(5 b B+A c\right)\sqrt{b x^2+c x^4}}{5 b x^{3/2}}-\frac{2 A\left(b x^2+c x^4\right)^{3/2}}{5 b x^{11/2}}-$$
$$\left(4 c^{1/4}\left(5 b B+A c\right) x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(5 b^{3/4}\sqrt{b x^2+c x^4}\right)+$$
$$\left(2 c^{1/4}\left(5 b B+A c\right) x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(5 b^{3/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 219 leaves):

$$\left(2\left(\sqrt{b}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(-A+5Bx^2\right)\left(b+cx^2\right)-\right.\right.$$

$$2\sqrt{c}\,(5bB+Ac)\sqrt{1+\frac{b}{cx^2}}\,x^{7/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+$$

$$\left.\left.2\sqrt{c}\,(5bB+Ac)\sqrt{1+\frac{b}{cx^2}}\,x^{7/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/$$

$$\left(5\sqrt{b}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,x^{3/2}\sqrt{x^2\left(b+cx^2\right)}\right)$$

## Problem 228: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$-\frac{2\,(7bB-Ac)\sqrt{bx^2+cx^4}}{21\,b\,x^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7\,b\,x^{13/2}}+$$

$$\left(2c^{3/4}\,(7bB-Ac)\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(21\,b^{5/4}\sqrt{bx^2+cx^4}\right)$$

Result (type 4, 138 leaves):

$$\frac{1}{21}\sqrt{x^2\left(b+cx^2\right)}$$

$$\left(-\frac{2\left(3Ab+7bBx^2+2Acx^2\right)}{b\,x^{9/2}}+\frac{4\,i\,c\,(7bB-Ac)\sqrt{1+\frac{b}{cx^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{b\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+cx^2\right)}\right)$$

## Problem 229: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \sqrt{b x^2+c x^4}}{x^{13/2}} \, dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$\frac{4 c^{3/2} (3 b B-A c) x^{3/2} \left(b+c x^2\right)}{15 b^2 \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{b x^2+c x^4}}-\frac{2 (3 b B-A c) \sqrt{b x^2+c x^4}}{15 b x^{7/2}}-\frac{4 c (3 b B-A c) \sqrt{b x^2+c x^4}}{15 b^2 x^{3/2}}-\frac{2 A \left(b x^2+c x^4\right)^{3/2}}{9 b x^{15/2}}-\left(4 c^{5/4} (3 b B-A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(15 b^{7/4} \sqrt{b x^2+c x^4}\right)+\left(2 c^{5/4} (3 b B-A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(15 b^{7/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 241 leaves):

$$-\left(\left(2 \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \left(b+c x^2\right) \left(9 b B x^2 \left(b+2 c x^2\right)+A \left(5 b^2+2 b c x^2-6 c^2 x^4\right)\right)-6 \sqrt{b} c^{3/2} (3 b B-A c) x^5 \sqrt{1+\frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]+6 \sqrt{b} c^{3/2} (3 b B-A c) x^5 \sqrt{1+\frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]\right)\right) \Big/ \left(45 b^2 x^{7/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)\right)$$

## Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \sqrt{b x^2+c x^4}}{x^{15/2}} \, dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$-\frac{2\,(11\,b\,B-5\,A\,c)\,\sqrt{b\,x^2+c\,x^4}}{77\,b\,x^{9/2}}-\frac{4\,c\,(11\,b\,B-5\,A\,c)\,\sqrt{b\,x^2+c\,x^4}}{231\,b^2\,x^{5/2}}-\frac{2\,A\,\left(b\,x^2+c\,x^4\right)^{3/2}}{11\,b\,x^{17/2}}-\left(2\,c^{7/4}\,(11\,b\,B-5\,A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(231\,b^{9/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 158 leaves):

$$\frac{1}{231\,b^2}\,2\,\sqrt{x^2\,\left(b+c\,x^2\right)}\left(\frac{-11\,b\,B\,x^2\,\left(3\,b+2\,c\,x^2\right)+A\,\left(-21\,b^2-6\,b\,c\,x^2+10\,c^2\,x^4\right)}{x^{13/2}}+\left(2\,i\,c^2\,(-11\,b\,B+5\,A\,c)\sqrt{1+\frac{b}{c\,x^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,\left(b+c\,x^2\right)\right)\right)$$

## Problem 231: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{7/2}\,\left(A+B\,x^2\right)\,\left(b\,x^2+c\,x^4\right)^{3/2}\,dx$$

Optimal (type 4, 486 leaves, 11 steps):

$$\frac{88\,b^5\,(3\,b\,B-5\,A\,c)\,x^{3/2}\,(b+c\,x^2)}{16575\,c^{9/2}\,(\sqrt{b}+\sqrt{c}\,x)\,\sqrt{b\,x^2+c\,x^4}}-\frac{88\,b^4\,(3\,b\,B-5\,A\,c)\,\sqrt{x}\,\sqrt{b\,x^2+c\,x^4}}{49725\,c^4}+$$
$$\frac{88\,b^3\,(3\,b\,B-5\,A\,c)\,x^{5/2}\,\sqrt{b\,x^2+c\,x^4}}{69615\,c^3}-\frac{8\,b^2\,(3\,b\,B-5\,A\,c)\,x^{9/2}\,\sqrt{b\,x^2+c\,x^4}}{7735\,c^2}-$$
$$\frac{4\,b\,(3\,b\,B-5\,A\,c)\,x^{13/2}\,\sqrt{b\,x^2+c\,x^4}}{595\,c}-\frac{2\,(3\,b\,B-5\,A\,c)\,x^{9/2}\,(b\,x^2+c\,x^4)^{3/2}}{105\,c}+\frac{2\,B\,x^{5/2}\,(b\,x^2+c\,x^4)^{5/2}}{25\,c}-$$
$$\left(88\,b^{21/4}\,(3\,b\,B-5\,A\,c)\,x\,(\sqrt{b}+\sqrt{c}\,x)\,\sqrt{\frac{b+c\,x^2}{(\sqrt{b}+\sqrt{c}\,x)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16575\,c^{19/4}\,\sqrt{b\,x^2+c\,x^4}\right)+$$
$$\left(44\,b^{21/4}\,(3\,b\,B-5\,A\,c)\,x\,(\sqrt{b}+\sqrt{c}\,x)\,\sqrt{\frac{b+c\,x^2}{(\sqrt{b}+\sqrt{c}\,x)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16575\,c^{19/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 332 leaves):

$$\frac{1}{348075\,c^5\,x^3\,(b+c\,x^2)^2}\,2\,\left(x^2\,(b+c\,x^2)\right)^{3/2}\left(\frac{1}{\sqrt{x}}\right.$$
$$(b+c\,x^2)\,\left(2772\,b^6\,B-924\,b^5\,c\,(5\,A+B\,x^2)+220\,b^4\,c^2\,x^2\,(7\,A+3\,B\,x^2)+36\,b^2\,c^4\,x^6\,(25\,A+13\,B\,x^2)+\right.$$
$$\left.663\,c^6\,x^{10}\,(25\,A+21\,B\,x^2)-20\,b^3\,c^3\,x^4\,(55\,A+27\,B\,x^2)+39\,b\,c^5\,x^8\,(575\,A+459\,B\,x^2)\right)+$$
$$924\,i\,b^5\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,c\,(3\,b\,B-5\,A\,c)\,\sqrt{1+\frac{b}{c\,x^2}}\,x\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-$$
$$\left.924\,i\,b^5\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,c\,(3\,b\,B-5\,A\,c)\,\sqrt{1+\frac{b}{c\,x^2}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)$$

## Problem 232: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2}\,(A+B\,x^2)\,(b\,x^2+c\,x^4)^{3/2}\,dx$$

Optimal (type 4, 321 leaves, 9 steps):

$$-\frac{24\,b^4\,(13\,b\,B-23\,A\,c)\,\sqrt{b\,x^2+c\,x^4}}{33649\,c^4\,\sqrt{x}}+\frac{72\,b^3\,(13\,b\,B-23\,A\,c)\,x^{3/2}\,\sqrt{b\,x^2+c\,x^4}}{168245\,c^3}-\frac{8\,b^2\,(13\,b\,B-23\,A\,c)\,x^{7/2}\,\sqrt{b\,x^2+c\,x^4}}{24035\,c^2}-\frac{4\,b\,(13\,b\,B-23\,A\,c)\,x^{11/2}\,\sqrt{b\,x^2+c\,x^4}}{2185\,c}-\frac{2\,(13\,b\,B-23\,A\,c)\,x^{7/2}\,(b\,x^2+c\,x^4)^{3/2}}{437\,c}+\frac{2\,B\,x^{3/2}\,(b\,x^2+c\,x^4)^{5/2}}{23\,c}+\left(12\,b^{19/4}\,(13\,b\,B-23\,A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(33649\,c^{17/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 219 leaves):

$$\frac{1}{168245\,c^4}2\,\sqrt{x^2\,(b+c\,x^2)}\left(\frac{1}{\sqrt{x}}\left(-780\,b^5\,B+28\,b^2\,c^3\,x^4\,(23\,A+11\,B\,x^2)+385\,c^5\,x^8\,(23\,A+19\,B\,x^2)+12\,b^4\,c\,(115\,A+39\,B\,x^2)-4\,b^3\,c^2\,x^2\,(207\,A+91\,B\,x^2)+77\,b\,c^4\,x^6\,(161\,A+125\,B\,x^2)\right)+\left(60\,i\,b^5\,(13\,b\,B-23\,A\,c)\,\sqrt{1+\frac{b}{c\,x^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,(b+c\,x^2)\right)\right)$$

## Problem 233: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2}\,(A+B\,x^2)\,(b\,x^2+c\,x^4)^{3/2}\,dx$$

Optimal (type 4, 447 leaves, 10 steps):

$$-\frac{8 b^4 (11 b B-21 A c) x^{3/2} (b+c x^2)}{3315 c^{7/2} (\sqrt{b}+\sqrt{c} x) \sqrt{b x^2+c x^4}}+\frac{8 b^3 (11 b B-21 A c) \sqrt{x} \sqrt{b x^2+c x^4}}{9945 c^3}-\frac{8 b^2 (11 b B-21 A c) x^{5/2} \sqrt{b x^2+c x^4}}{13923 c^2}-\frac{4 b (11 b B-21 A c) x^{9/2} \sqrt{b x^2+c x^4}}{1547 c}-\frac{2 (11 b B-21 A c) x^{5/2} (b x^2+c x^4)^{3/2}}{357 c}+\frac{2 B \sqrt{x} (b x^2+c x^4)^{5/2}}{21 c}+\left(8 b^{17/4} (11 b B-21 A c) x (\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(3315 c^{15/4} \sqrt{b x^2+c x^4}\right)-\left(4 b^{17/4} (11 b B-21 A c) x (\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(3315 c^{15/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 313 leaves):

$$\left(2 (x^2 (b+c x^2))^{3/2} \left(c x^{3/2} (b+c x^2) \left(28 b^3 (11 b B-21 A c)-20 b^2 c (11 b B-21 A c) x^2+45 b c^2 (4 b B+133 A c) x^4+195 c^3 (23 b B+21 A c) x^6+3315 B c^4 x^8\right)-\frac{1}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x}} 84 b^4 (11 b B-21 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b+c x^2)-\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right)\right) \Big/ \left(69615 c^4 x^3 (b+c x^2)^2\right)$$

## Problem 234: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x}\,\left(A+B x^2\right)\left(b x^2+c x^4\right)^{3/2}\,dx$$

Optimal (type 4, 282 leaves, 8 steps):

$$\frac{8 b^3 (9 b B-19 A c)\sqrt{b x^2+c x^4}}{4389 c^3 \sqrt{x}}-\frac{8 b^2 (9 b B-19 A c)\, x^{3/2}\sqrt{b x^2+c x^4}}{7315 c^2}-\frac{4 b (9 b B-19 A c)\, x^{7/2}\sqrt{b x^2+c x^4}}{1045 c}-\frac{2 (9 b B-19 A c)\, x^{3/2}\left(b x^2+c x^4\right)^{3/2}}{285 c}+\frac{2 B\left(b x^2+c x^4\right)^{5/2}}{19 c \sqrt{x}}-\left(4 b^{15/4} (9 b B-19 A c)\, x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4389 c^{13/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 198 leaves):

$$\frac{1}{21945 c^3}2\sqrt{x^2\left(b+c x^2\right)}\left(\frac{1}{\sqrt{x}}\left(180 b^4 B+12 b^2 c^2 x^2\left(19 A+7 B x^2\right)+77 c^4 x^6\left(19 A+15 B x^2\right)-4 b^3 c\left(95 A+27 B x^2\right)+7 b c^3 x^4\left(323 A+231 B x^2\right)\right)+\left(20\, i\, b^4(-9 b B+19 A c)\sqrt{1+\frac{b}{c x^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)\right)\right)$$

## Problem 235: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right)\left(b x^2+c x^4\right)^{3/2}}{\sqrt{x}}\,dx$$

Optimal (type 4, 408 leaves, 9 steps):

$$\frac{8 b^3 (7 b B-17 A c) x^{3/2} (b+c x^2)}{1105 c^{5/2} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}-\frac{8 b^2 (7 b B-17 A c) \sqrt{x} \sqrt{b x^2+c x^4}}{3315 c^2}-$$
$$\frac{4 b (7 b B-17 A c) x^{5/2} \sqrt{b x^2+c x^4}}{663 c}-\frac{2 (7 b B-17 A c) \sqrt{x} (b x^2+c x^4)^{3/2}}{221 c}+\frac{2 B (b x^2+c x^4)^{5/2}}{17 c x^{3/2}}-$$
$$\left(8 b^{13/4} (7 b B-17 A c) x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(1105 c^{11/4} \sqrt{b x^2+c x^4}\right)+$$
$$\left(4 b^{13/4} (7 b B-17 A c) x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(1105 c^{11/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 303 leaves):

$$\frac{1}{1105 x^3} \left(x^2 (b+c x^2)\right)^{3/2} \left(\frac{1}{3 c^2 (b+c x^2)}\right.$$
$$2 x^{3/2} \left(-28 b^3 B+4 b^2 c (17 A+5 B x^2)+15 c^3 x^4 (17 A+13 B x^2)+5 b c^2 x^2 (85 A+57 B x^2)\right)+$$
$$\left(8 b^3 (7 b B-17 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b+c x^2)-\right.\right.$$
$$\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}}$$
$$\left.\left.\left. x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^3 \sqrt{x} (b+c x^2)^2\right)\right)$$

## Problem 236: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x^2) (b x^2+c x^4)^{3/2}}{x^{3/2}} \, dx$$

Optimal (type 4, 239 leaves, 7 steps):

$$-\frac{8 b^2 (b B-3 A c) \sqrt{b x^2+c x^4}}{231 c^2 \sqrt{x}}-\frac{4 b (b B-3 A c) x^{3/2} \sqrt{b x^2+c x^4}}{77 c}-\frac{2 (b B-3 A c) \left(b x^2+c x^4\right)^{3/2}}{33 c \sqrt{x}}+\frac{2 B \left(b x^2+c x^4\right)^{5/2}}{15 c x^{5/2}}+\left(4 b^{11/4} (b B-3 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(231 c^{9/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 174 leaves):

$$\frac{1}{1155 c^2} 2 \sqrt{x^2 \left(b+c x^2\right)} \left(\frac{1}{\sqrt{x}}\left(-20 b^3 B+12 b^2 c \left(5 A+B x^2\right)+7 c^3 x^4 \left(15 A+11 B x^2\right)+b c^2 x^2 \left(195 A+119 B x^2\right)\right)+\frac{20 i b^3 (b B-3 A c) \sqrt{1+\frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \left(b+c x^2\right)}\right)$$

## Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right) \left(b x^2+c x^4\right)^{3/2}}{x^{5/2}} \, dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$-\frac{8 b^2 (3 b B-13 A c) x^{3/2} (b+c x^2)}{195 c^{3/2} (\sqrt{b}+\sqrt{c} x) \sqrt{b x^2+c x^4}}-\frac{4 b (3 b B-13 A c) \sqrt{x} \sqrt{b x^2+c x^4}}{195 c}-\frac{2 (3 b B-13 A c) (b x^2+c x^4)^{3/2}}{117 c x^{3/2}}+\frac{2 B (b x^2+c x^4)^{5/2}}{13 c x^{7/2}}+\left(8 b^{9/4} (3 b B-13 A c) x (\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(195 c^{7/4} \sqrt{b x^2+c x^4}\right)-\left(4 b^{9/4} (3 b B-13 A c) x (\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(195 c^{7/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 281 leaves) :

$$\frac{1}{195 x^3} (x^2 (b+c x^2))^{3/2} \left(\frac{2 x^{3/2} (12 b^2 B+5 c^2 x^2 (13 A+9 B x^2)+b c (143 A+75 B x^2))}{3 c (b+c x^2)}-\left(8 b^2 (3 b B-13 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b+c x^2)-\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 \sqrt{x} (b+c x^2)^2\right)\right)$$

## Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x^2) (b x^2+c x^4)^{3/2}}{x^{7/2}} \, dx$$

Optimal (type 4, 201 leaves, 6 steps) :

$$-\frac{4\,b\,(b\,B-11\,A\,c)\,\sqrt{b\,x^2+c\,x^4}}{77\,c\,\sqrt{x}}-\frac{2\,(b\,B-11\,A\,c)\,\left(b\,x^2+c\,x^4\right)^{3/2}}{77\,c\,x^{5/2}}+\frac{2\,B\,\left(b\,x^2+c\,x^4\right)^{5/2}}{11\,c\,x^{9/2}}-$$
$$\left(4\,b^{7/4}\,(b\,B-11\,A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$
$$\left(77\,c^{5/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 153 leaves) :

$$\frac{1}{77\,c}2\,\sqrt{x^2\,\left(b+c\,x^2\right)}\left(\frac{4\,b^2\,B+c^2\,x^2\,\left(11\,A+7\,B\,x^2\right)+b\,c\,\left(33\,A+13\,B\,x^2\right)}{\sqrt{x}}-\right.$$
$$\left.\frac{4\,i\,b^2\,(b\,B-11\,A\,c)\,\sqrt{1+\frac{b}{c\,x^2}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]}{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\;\left(b+c\,x^2\right)}\right)$$

## Problem 239: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(b\,x^2+c\,x^4\right)^{3/2}}{x^{9/2}}\,dx$$

Optimal (type 4, 356 leaves, 8 steps) :

$$\frac{8\,b\,(b\,B+9\,A\,c)\,x^{3/2}\,(b+c\,x^2)}{15\,\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}+\frac{4}{15}\,(b\,B+9\,A\,c)\,\sqrt{x}\,\sqrt{b\,x^2+c\,x^4}+$$
$$\frac{2\,(b\,B+9\,A\,c)\,(b\,x^2+c\,x^4)^{3/2}}{9\,b\,x^{3/2}}-\frac{2\,A\,(b\,x^2+c\,x^4)^{5/2}}{b\,x^{11/2}}-$$
$$\left(8\,b^{5/4}\,(b\,B+9\,A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(15\,c^{3/4}\,\sqrt{b\,x^2+c\,x^4}\right)+$$
$$\left(4\,b^{5/4}\,(b\,B+9\,A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(15\,c^{3/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 249 leaves):

$$\left(2\,\sqrt{x}\,\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,(b+c\,x^2)\,\left(12\,b^2\,B+c^2\,x^2\,(9\,A+5\,B\,x^2)+b\,c\,(63\,A+11\,B\,x^2)\right)-\right.\right.$$
$$12\,b^{3/2}\,\sqrt{c}\,(b\,B+9\,A\,c)\,\sqrt{1+\frac{b}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+$$
$$\left.\left.12\,b^{3/2}\,\sqrt{c}\,(b\,B+9\,A\,c)\,\sqrt{1+\frac{b}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/$$
$$\left(45\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,c\,\sqrt{x^2\,(b+c\,x^2)}\right)$$

## Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(A+B\,x^2)\,(b\,x^2+c\,x^4)^{3/2}}{x^{11/2}}\,dx$$

Optimal (type 4, 200 leaves, 6 steps):

$$\frac{4\,(3\,b\,B+7\,A\,c)\,\sqrt{b\,x^2+c\,x^4}}{21\,\sqrt{x}}+\frac{2\,(3\,b\,B+7\,A\,c)\,\left(b\,x^2+c\,x^4\right)^{3/2}}{21\,b\,x^{5/2}}-\frac{2\,A\,\left(b\,x^2+c\,x^4\right)^{5/2}}{3\,b\,x^{13/2}}+\left(4\,b^{3/4}\,(3\,b\,B+7\,A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(21\,c^{1/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 138 leaves):

$$\frac{2}{21}\sqrt{x^2\,\left(b+c\,x^2\right)}\left(\frac{-7\,A\,b+9\,b\,B\,x^2+7\,A\,c\,x^2+3\,B\,c\,x^4}{x^{5/2}}+\frac{4\,i\,b\,(3\,b\,B+7\,A\,c)\,\sqrt{1+\frac{b}{c\,x^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,\left(b+c\,x^2\right)}\right)$$

## Problem 241: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\,\left(b\,x^2+c\,x^4\right)^{3/2}}{x^{13/2}}\,dx$$

Optimal (type 4, 354 leaves, 8 steps):

$$\frac{24\,\sqrt{c}\,(b\,B+A\,c)\,x^{3/2}\,\left(b+c\,x^2\right)}{5\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}+\frac{12\,c\,(b\,B+A\,c)\,\sqrt{x}\,\sqrt{b\,x^2+c\,x^4}}{5\,b}-\frac{2\,(b\,B+A\,c)\,\left(b\,x^2+c\,x^4\right)^{3/2}}{b\,x^{7/2}}-\frac{2\,A\,\left(b\,x^2+c\,x^4\right)^{5/2}}{5\,b\,x^{15/2}}-\frac{1}{5\,\sqrt{b\,x^2+c\,x^4}}24\,b^{1/4}\,c^{1/4}\,(b\,B+A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{5\,\sqrt{b\,x^2+c\,x^4}}12\,b^{1/4}\,c^{1/4}\,(b\,B+A\,c)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 232 leaves):

$$\left(2\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)\left(-A b+7 b B x^2+5 A c x^2+B c x^4\right)-\right.\right.$$

$$12\sqrt{b}\sqrt{c}\left(b B+A c\right)\sqrt{1+\frac{b}{c x^2}}\, x^{7/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+$$

$$\left.\left.12\sqrt{b}\sqrt{c}\left(b B+A c\right)\sqrt{1+\frac{b}{c x^2}}\, x^{7/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/$$

$$\left(5\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, x^{3/2}\sqrt{x^2\left(b+c x^2\right)}\right)$$

## Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(A+B x^2\right)\left(b x^2+c x^4\right)^{3/2}}{x^{15/2}}\, dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$\frac{4 c\left(7 b B+3 A c\right)\sqrt{b x^2+c x^4}}{21 b\sqrt{x}}-\frac{2\left(7 b B+3 A c\right)\left(b x^2+c x^4\right)^{3/2}}{21 b x^{9/2}}-\frac{2 A\left(b x^2+c x^4\right)^{5/2}}{7 b x^{17/2}}+$$

$$\left(4 c^{3/4}\left(7 b B+3 A c\right) x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2\, \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(21 b^{1/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 139 leaves):

$$\frac{2}{21}\sqrt{x^2\left(b+c\,x^2\right)}\left(\frac{7\,B\,x^2\left(-b+c\,x^2\right)-3\,A\left(b+3\,c\,x^2\right)}{x^{9/2}}+\frac{4\,\mathrm{i}\,c\left(7\,b\,B+3\,A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\left(b+c\,x^2\right)}\right)$$

## Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(A+B\,x^2\right)\left(b\,x^2+c\,x^4\right)^{3/2}}{x^{17/2}}\,dx$$

Optimal (type 4, 364 leaves, 8 steps):

$$\frac{8\,c^{3/2}\left(9\,b\,B+A\,c\right)x^{3/2}\left(b+c\,x^2\right)}{15\,b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}-\frac{4\,c\left(9\,b\,B+A\,c\right)\sqrt{b\,x^2+c\,x^4}}{15\,b\,x^{3/2}}-\frac{2\left(9\,b\,B+A\,c\right)\left(b\,x^2+c\,x^4\right)^{3/2}}{45\,b\,x^{11/2}}-\frac{2\,A\left(b\,x^2+c\,x^4\right)^{5/2}}{9\,b\,x^{19/2}}-\left(8\,c^{5/4}\left(9\,b\,B+A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(15\,b^{3/4}\sqrt{b\,x^2+c\,x^4}\right)+\left(4\,c^{5/4}\left(9\,b\,B+A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(15\,b^{3/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 236 leaves):

$$-\left(\left(\left(2\sqrt{b}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)\left(9 B x^2\left(b-5 c x^2\right)+A\left(5 b+11 c x^2\right)\right)+\right.\right.\right.$$

$$12 c^{3/2}(9 b B+A c)\sqrt{1+\frac{b}{c x^2}}\, x^{11/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-$$

$$\left.\left.12 c^{3/2}(9 b B+A c)\sqrt{1+\frac{b}{c x^2}}\, x^{11/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/$$

$$\left.\left(45\sqrt{b}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, x^{7/2}\sqrt{x^2\left(b+c x^2\right)}\right)\right)$$

## Problem 244: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^{13/2}\left(A+B x^2\right)}{\sqrt{b x^2+c x^4}}\,dx$$

Optimal (type 4, 243 leaves, 7 steps):

$$-\frac{2 b^2(13 b B-15 A c)\sqrt{b x^2+c x^4}}{77 c^4\sqrt{x}}+\frac{6 b(13 b B-15 A c)x^{3/2}\sqrt{b x^2+c x^4}}{385 c^3}-$$

$$\frac{2(13 b B-15 A c)x^{7/2}\sqrt{b x^2+c x^4}}{165 c^2}+\frac{2 B x^{11/2}\sqrt{b x^2+c x^4}}{15 c}+$$

$$\left(b^{11/4}(13 b B-15 A c)\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(77 c^{17/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 196 leaves):

$$\left(-2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,x^{3/2}\left(b+c\,x^2\right)\left(195\,b^3\,B-7\,c^3\,x^4\left(15\,A+11\,B\,x^2\right)-9\,b^2\,c\left(25\,A+13\,B\,x^2\right)+b\,c^2\,x^2\left(135\,A+91\,B\,x^2\right)\right)-30\,i\,b^3\left(-13\,b\,B+15\,A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(1155\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,c^4\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

## Problem 245: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^{11/2}\left(A+B\,x^2\right)}{\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$-\frac{14\,b^2\left(11\,b\,B-13\,A\,c\right)x^{3/2}\left(b+c\,x^2\right)}{195\,c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}+\frac{14\,b\left(11\,b\,B-13\,A\,c\right)\sqrt{x}\,\sqrt{b\,x^2+c\,x^4}}{585\,c^3}-\frac{2\left(11\,b\,B-13\,A\,c\right)x^{5/2}\sqrt{b\,x^2+c\,x^4}}{117\,c^2}+\frac{2\,B\,x^{9/2}\sqrt{b\,x^2+c\,x^4}}{13\,c}+\left(14\,b^{9/4}\left(11\,b\,B-13\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(195\,c^{15/4}\sqrt{b\,x^2+c\,x^4}\right)-\left(7\,b^{9/4}\left(11\,b\,B-13\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(195\,c^{15/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 264 leaves):

$$\left(2 x \left(c x^{3/2} \left(b+c x^2\right) \left(77 b^2 B+5 c^2 x^2 \left(13 A+9 B x^2\right)-b c \left(91 A+55 B x^2\right)\right)-\frac{1}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x}} 21 b^2 \left(11 b B-13 A c\right) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \left(b+c x^2\right)-\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\sqrt{b} \sqrt{c} \sqrt{1+\frac{b}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\right) \Big/ \left(585 c^4 \sqrt{x^2 \left(b+c x^2\right)}\right)$$

## Problem 246: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2} \left(A+B x^2\right)}{\sqrt{b x^2+c x^4}} \, dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$\frac{10 b \left(9 b B-11 A c\right) \sqrt{b x^2+c x^4}}{231 c^3 \sqrt{x}}-\frac{2 \left(9 b B-11 A c\right) x^{3/2} \sqrt{b x^2+c x^4}}{77 c^2}+\frac{2 B x^{7/2} \sqrt{b x^2+c x^4}}{11 c}-\left(5 b^{7/4} \left(9 b B-11 A c\right) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(231 c^{13/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 176 leaves):

$$\left(2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,x^{3/2}\left(b+c\,x^2\right)\left(45\,b^2\,B+3\,c^2\,x^2\left(11\,A+7\,B\,x^2\right)-b\,c\left(55\,A+27\,B\,x^2\right)\right)+\right.$$

$$\left.10\,i\,b^2\left(-9\,b\,B+11\,A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Bigg/$$

$$\left(231\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,c^3\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

## Problem 247: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^{7/2}\left(A+B\,x^2\right)}{\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 330 leaves, 7 steps):

$$\frac{2\,b\left(7\,b\,B-9\,A\,c\right)x^{3/2}\left(b+c\,x^2\right)}{15\,c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}-\frac{2\left(7\,b\,B-9\,A\,c\right)\sqrt{x}\,\sqrt{b\,x^2+c\,x^4}}{45\,c^2}+\frac{2\,B\,x^{5/2}\sqrt{b\,x^2+c\,x^4}}{9\,c}-$$

$$\left(2\,b^{5/4}\left(7\,b\,B-9\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/$$

$$\left(15\,c^{11/4}\sqrt{b\,x^2+c\,x^4}\right)+$$

$$\left(b^{5/4}\left(7\,b\,B-9\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/$$

$$\left(15\,c^{11/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 237 leaves):

$$\left(2\sqrt{x}\,\left(b+c\,x^2\right)\,\left(21\,b^2\,B+c^2\,x^2\,\left(9\,A+5\,B\,x^2\right)-b\,c\,\left(27\,A+7\,B\,x^2\right)\right)+6\,i\,b\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,c\,\left(7\,b\,B-9\,A\,c\right)\,\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-6\,i\,b\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,c\,\left(7\,b\,B-9\,A\,c\right)\,\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(45\,c^3\,\sqrt{x^2\,\left(b+c\,x^2\right)}\right)$$

## Problem 248: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^{5/2}\,\left(A+B\,x^2\right)}{\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$-\frac{2\,\left(5\,b\,B-7\,A\,c\right)\,\sqrt{b\,x^2+c\,x^4}}{21\,c^2\,\sqrt{x}}+\frac{2\,B\,x^{3/2}\,\sqrt{b\,x^2+c\,x^4}}{7\,c}+\left(b^{3/4}\,\left(5\,b\,B-7\,A\,c\right)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\,\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(21\,c^{9/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 151 leaves):

$$\left(-2\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,x^{3/2}\,\left(b+c\,x^2\right)\,\left(5\,b\,B-7\,A\,c-3\,B\,c\,x^2\right)-2\,i\,b\,\left(-5\,b\,B+7\,A\,c\right)\,\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(21\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{c}}}\,c^2\,\sqrt{x^2\,\left(b+c\,x^2\right)}\right)$$

## Problem 249: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}\left(A+B x^2\right)}{\sqrt{b x^2+c x^4}}\,dx$$

Optimal (type 4, 293 leaves, 6 steps):

$$-\frac{2\left(3 b B-5 A c\right) x^{3/2}\left(b+c x^2\right)}{5 c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b x^2+c x^4}}+\frac{2 B \sqrt{x}\,\sqrt{b x^2+c x^4}}{5 c}+\left(2 b^{1/4}\left(3 b B-5 A c\right) x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(5 c^{7/4}\sqrt{b x^2+c x^4}\right)-\left(b^{1/4}\left(3 b B-5 A c\right) x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(5 c^{7/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 209 leaves):

$$\left(2 x\left(\frac{\left(b+c x^2\right)\left(-3 b B+5 A c+B c x^2\right)}{c\sqrt{x}}-i\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(3 b B-5 A c\right)\sqrt{1+\frac{b}{c x^2}}\,x\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+i\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(3 b B-5 A c\right)\sqrt{1+\frac{b}{c x^2}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(5 c\sqrt{x^2\left(b+c x^2\right)}\right)$$

## Problem 250: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}\left(A+B x^2\right)}{\sqrt{b x^2+c x^4}}\,dx$$

Optimal (type 4, 130 leaves, 4 steps):

$$\frac{2 B \sqrt{b x^2+c x^4}}{3 c \sqrt{x}}-\left((b B-3 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(3 b^{1/4} c^{5/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 134 leaves):

$$\frac{2 B x^{3/2} \left(b+c x^2\right)}{3 c \sqrt{x^2 \left(b+c x^2\right)}}-\frac{2 i (b B-3 A c) \sqrt{1+\frac{b}{c x^2}} x^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{3 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c \sqrt{x^2 \left(b+c x^2\right)}}$$

## Problem 251: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{\sqrt{x} \sqrt{b x^2+c x^4}} \, dx$$

Optimal (type 4, 281 leaves, 6 steps):

$$\frac{2 (b B+A c) x^{3/2} \left(b+c x^2\right)}{b \sqrt{c} \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{b x^2+c x^4}}-\frac{2 A \sqrt{b x^2+c x^4}}{b x^{3/2}}-\left(2 (b B+A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(b^{3/4} c^{3/4} \sqrt{b x^2+c x^4}\right)+\left((b B+A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(b^{3/4} c^{3/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 191 leaves):

$$-\left(\left(2\,\mathrm{i}\,x^{3/2}\left(A\sqrt{c}\,\sqrt{\frac{\mathrm{i}\sqrt{c}\,x}{\sqrt{b}}}\,\left(b+c\,x^2\right)-\right.\right.\right.$$
$$\sqrt{b}\,\left(b\,B+A\,c\right)\,x\,\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+$$
$$\left.\left.\sqrt{b}\,\left(b\,B+A\,c\right)\,x\,\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/$$
$$\left.\left(b^{3/2}\left(\frac{\mathrm{i}\sqrt{c}\,x}{\sqrt{b}}\right)^{3/2}\sqrt{x^2\left(b+c\,x^2\right)}\right)\right)$$

## Problem 252: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{x^{3/2}\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$-\frac{2\,A\sqrt{b\,x^2+c\,x^4}}{3\,b\,x^{5/2}}+$$
$$\left(\left(3\,b\,B-A\,c\right)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(3\,b^{5/4}\,c^{1/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 119 leaves):

$$\frac{2\left(-A\left(b+c\,x^2\right)+\frac{\mathrm{i}\,(3\,b\,B-A\,c)\sqrt{1+\frac{b}{c\,x^2}}\,x^{5/2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}\right)}{3\,b\sqrt{x}\,\sqrt{x^2\left(b+c\,x^2\right)}}$$

## Problem 253: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{x^{5/2}\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 332 leaves, 7 steps):

$$\frac{2\sqrt{c}\,(5\,b\,B-3\,A\,c)\,x^{3/2}\,(b+c\,x^2)}{5\,b^2\,(\sqrt{b}+\sqrt{c}\,x)\,\sqrt{b\,x^2+c\,x^4}}-\frac{2\,A\,\sqrt{b\,x^2+c\,x^4}}{5\,b\,x^{7/2}}-\frac{2\,(5\,b\,B-3\,A\,c)\,\sqrt{b\,x^2+c\,x^4}}{5\,b^2\,x^{3/2}}-$$
$$\left(2\,c^{1/4}\,(5\,b\,B-3\,A\,c)\,x\,(\sqrt{b}+\sqrt{c}\,x)\,\sqrt{\frac{b+c\,x^2}{(\sqrt{b}+\sqrt{c}\,x)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(5\,b^{7/4}\,\sqrt{b\,x^2+c\,x^4}\right)+$$
$$\left(c^{1/4}\,(5\,b\,B-3\,A\,c)\,x\,(\sqrt{b}+\sqrt{c}\,x)\,\sqrt{\frac{b+c\,x^2}{(\sqrt{b}+\sqrt{c}\,x)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(5\,b^{7/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 222 leaves):

$$\left(2\,\sqrt{b}\,\sqrt{c}\,(5\,b\,B-3\,A\,c)\,x^3\,\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]-\right.$$
$$2\left(\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\,(b+c\,x^2)\,\left(5\,b\,B\,x^2+A\,(b-3\,c\,x^2)\right)+\sqrt{b}\,\sqrt{c}\,(5\,b\,B-3\,A\,c)\,x^3\,\sqrt{1+\frac{c\,x^2}{b}}\right.$$
$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\right)\Big/\left(5\,b^2\,x^{3/2}\,\sqrt{\frac{i\,\sqrt{c}\,x}{\sqrt{b}}}\,\sqrt{x^2\,(b+c\,x^2)}\right)$$

## Problem 254: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{x^{7/2}\,\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$-\frac{2\,A\,\sqrt{b\,x^2+c\,x^4}}{7\,b\,x^{9/2}}-\frac{2\,(7\,b\,B-5\,A\,c)\,\sqrt{b\,x^2+c\,x^4}}{21\,b^2\,x^{5/2}}-$$
$$\left(c^{3/4}\,(7\,b\,B-5\,A\,c)\,x\,(\sqrt{b}+\sqrt{c}\,x)\,\sqrt{\frac{b+c\,x^2}{(\sqrt{b}+\sqrt{c}\,x)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(21\,b^{9/4}\,\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 156 leaves):

$$\left(-2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c x^2\right)\left(3 A b+7 b B x^2-5 A c x^2\right)+\right.$$
$$\left.2 i c\left(-7 b B+5 A c\right)\sqrt{1+\frac{b}{c x^2}}\, x^{9/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/$$
$$\left(21 b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, x^{5/2}\sqrt{x^2\left(b+c x^2\right)}\right)$$

## Problem 255: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^2}{x^{9/2}\sqrt{b x^2+c x^4}}\,dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$-\frac{2 c^{3/2}\left(9 b B-7 A c\right) x^{3/2}\left(b+c x^2\right)}{15 b^3\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{b x^2+c x^4}}-\frac{2 A\sqrt{b x^2+c x^4}}{9 b x^{11/2}}-$$
$$\frac{2\left(9 b B-7 A c\right)\sqrt{b x^2+c x^4}}{45 b^2 x^{7/2}}+\frac{2 c\left(9 b B-7 A c\right)\sqrt{b x^2+c x^4}}{15 b^3 x^{3/2}}+$$
$$\left(2 c^{5/4}\left(9 b B-7 A c\right) x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(15 b^{11/4}\sqrt{b x^2+c x^4}\right)-$$
$$\left(c^{5/4}\left(9 b B-7 A c\right) x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(15 b^{11/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 242 leaves):

$$\left(2\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(b+c\,x^2\right)\left(-9\,b\,B\,x^2\left(b-3\,c\,x^2\right)+A\left(-5\,b^2+7\,b\,c\,x^2-21\,c^2\,x^4\right)\right)-\right.$$
$$6\sqrt{b}\,c^{3/2}\left(9\,b\,B-7\,A\,c\right)x^5\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]+$$
$$\left.6\sqrt{b}\,c^{3/2}\left(9\,b\,B-7\,A\,c\right)x^5\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\Big/$$
$$\left(45\,b^3\,x^{7/2}\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

## Problem 256: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{x^{11/2}\sqrt{b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$-\frac{2\,A\sqrt{b\,x^2+c\,x^4}}{11\,b\,x^{13/2}}-\frac{2\left(11\,b\,B-9\,A\,c\right)\sqrt{b\,x^2+c\,x^4}}{77\,b^2\,x^{9/2}}+\frac{10\,c\left(11\,b\,B-9\,A\,c\right)\sqrt{b\,x^2+c\,x^4}}{231\,b^3\,x^{5/2}}+$$
$$\left(5\,c^{7/4}\left(11\,b\,B-9\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(231\,b^{13/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 181 leaves):

$$\left(-2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+c\,x^2\right)\left(11\,b\,B\,x^2\left(3\,b-5\,c\,x^2\right)+3\,A\left(7\,b^2-9\,b\,c\,x^2+15\,c^2\,x^4\right)\right)-\right.$$
$$\left.10\,i\,c^2\left(-11\,b\,B+9\,A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\,x^{13/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/$$
$$\left(231\,b^3\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,x^{9/2}\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

## Problem 257: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{17/2}\left(A+B x^2\right)}{\left(b x^2+c x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 251 leaves, 7 steps):

$$-\frac{(b B-A c)\, x^{15/2}}{b c \sqrt{b x^2+c x^4}}+\frac{15 b\,(13 b B-11 A c) \sqrt{b x^2+c x^4}}{77 c^4 \sqrt{x}}-\frac{9\,(13 b B-11 A c)\, x^{3/2} \sqrt{b x^2+c x^4}}{77 c^3}+\frac{(13 b B-11 A c)\, x^{7/2} \sqrt{b x^2+c x^4}}{11 b c^2}-\left(15 b^{7/4}\,(13 b B-11 A c)\, x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(154\, c^{17/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 189 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{3/2} \left(195 b^3 B+2 c^3 x^4 \left(11 A+7 B x^2\right)-2 b c^2 x^2 \left(33 A+13 B x^2\right)+b^2 \left(-165 A c+78 B c x^2\right)\right)+15\, i\, b^2\,(-13 b B+11 A c) \sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Big/ \left(77 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c^4 \sqrt{x^2 \left(b+c x^2\right)}\right)$$

## Problem 258: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{15/2}\left(A+B x^2\right)}{\left(b x^2+c x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 377 leaves, 8 steps):

$$-\frac{(b B-A c)\, x^{13/2}}{b c \sqrt{b x^2+c x^4}}+\frac{7 b (11 b B-9 A c)\, x^{3/2} (b+c x^2)}{15 c^{7/2} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}-\frac{7 (11 b B-9 A c) \sqrt{x} \sqrt{b x^2+c x^4}}{45 c^3}+\frac{(11 b B-9 A c)\, x^{5/2} \sqrt{b x^2+c x^4}}{9 b c^2}-$$

$$\left(7 b^{5/4} (11 b B-9 A c)\, x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(15 c^{15/4} \sqrt{b x^2+c x^4}\right)+$$

$$\left(7 b^{5/4} (11 b B-9 A c)\, x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(30 c^{15/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 263 leaves) :

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x} \left(231 b^3 B-7 b^2 c \left(27 A-22 B x^2\right)+2 c^3 x^4 \left(9 A+5 B x^2\right)-2 b c^2 x^2 \left(63 A+11 B x^2\right)\right)+\right.$$

$$21 b^{3/2} \sqrt{c}\, (-11 b B+9 A c) \sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]-$$

$$\left.21 b^{3/2} \sqrt{c}\, (-11 b B+9 A c) \sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Big/$$

$$\left(45 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c^4 \sqrt{x^2 \left(b+c x^2\right)}\right)$$

## Problem 259: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13/2} \left(A+B x^2\right)}{\left(b x^2+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 214 leaves, 6 steps) :

$$-\frac{(b B-A c)\, x^{11/2}}{b c \sqrt{b x^2+c x^4}}-\frac{5\,(9 b B-7 A c) \sqrt{b x^2+c x^4}}{21 c^3 \sqrt{x}}+\frac{(9 b B-7 A c)\, x^{3/2} \sqrt{b x^2+c x^4}}{7 b c^2}+\left(5 b^{3/4}\,(9 b B-7 A c)\, x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(42 c^{13/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 165 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, x^{3/2} \left(-45 b^2 B+b c \left(35 A-18 B x^2\right)+2 c^2 x^2 \left(7 A+3 B x^2\right)\right)-5 i b\,(-9 b B+7 A c) \sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Big/ \left(21 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\, c^3 \sqrt{x^2 \left(b+c x^2\right)}\right)$$

## Problem 260: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{11/2} \left(A+B x^2\right)}{\left(b x^2+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 340 leaves, 7 steps):

$$-\frac{(b B-A c)\, x^{9/2}}{b c \sqrt{b x^2+c x^4}}-\frac{3\,(7 b B-5 A c)\, x^{3/2} \left(b+c x^2\right)}{5 c^{5/2} \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{b x^2+c x^4}}+\frac{(7 b B-5 A c) \sqrt{x} \sqrt{b x^2+c x^4}}{5 b c^2}+\left(3 b^{1/4}\,(7 b B-5 A c)\, x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(5 c^{11/4} \sqrt{b x^2+c x^4}\right)-\left(3 b^{1/4}\,(7 b B-5 A c)\, x \left(\sqrt{b}+\sqrt{c}\, x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(10 c^{11/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 240 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x}\left(-21 b^2 B+b c\left(15 A-14 B x^2\right)+2 c^2 x^2\left(5 A+B x^2\right)\right)-3\sqrt{b}\sqrt{c}\left(-7 b B+5 A c\right)\sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+3\sqrt{b}\sqrt{c}\left(-7 b B+5 A c\right)\sqrt{1+\frac{b}{c x^2}}\, x^2\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(5\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, c^3\sqrt{x^2\left(b+c x^2\right)}\right)$$

## Problem 261: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2}\left(A+B x^2\right)}{\left(b x^2+c x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 178 leaves, 5 steps):

$$-\frac{\left(b B-A c\right) x^{7/2}}{b c\sqrt{b x^2+c x^4}}+\frac{\left(5 b B-3 A c\right)\sqrt{b x^2+c x^4}}{3 b c^2\sqrt{x}}-\left(\left(5 b B-3 A c\right) x\left(\sqrt{b}+\sqrt{c}\, x\right)\sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2\, \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(6\, b^{1/4} c^{9/4}\sqrt{b x^2+c x^4}\right)$$

Result (type 4, 142 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, x^{3/2} \left(5\, b\, B - 3\, A\, c + 2\, B\, c\, x^2\right) + i \left(-5\, b\, B + 3\, A\, c\right) \sqrt{1 + \frac{b}{c\, x^2}}\right.$$

$$\left. x^2\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Big/ \left(3 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\, c^2 \sqrt{x^2 \left(b + c\, x^2\right)}\right)$$

## Problem 262: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2} \left(A + B\, x^2\right)}{\left(b\, x^2 + c\, x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 299 leaves, 6 steps):

$$-\frac{\left(b\, B - A\, c\right) x^{5/2}}{b\, c \sqrt{b\, x^2 + c\, x^4}} + \frac{\left(3\, b\, B - A\, c\right) x^{3/2} \left(b + c\, x^2\right)}{b\, c^{3/2} \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{b\, x^2 + c\, x^4}} -$$

$$\left(\left(3\, b\, B - A\, c\right) x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b + c\, x^2}{\left(\sqrt{b} + \sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2\, \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(b^{3/4}\, c^{7/4} \sqrt{b\, x^2 + c\, x^4}\right) +$$

$$\left(\left(3\, b\, B - A\, c\right) x \left(\sqrt{b} + \sqrt{c}\, x\right) \sqrt{\frac{b + c\, x^2}{\left(\sqrt{b} + \sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2\, \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(2\, b^{3/4}\, c^{7/4} \sqrt{b\, x^2 + c\, x^4}\right)$$

Result (type 4, 213 leaves):

$$\left(\mathrm{i}\left(\sqrt{b}\,\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\,\sqrt{x}\,\left(3\,b\,B-A\,c+2\,B\,c\,x^2\right)+\sqrt{c}\,\left(-3\,b\,B+A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-\sqrt{c}\,\left(-3\,b\,B+A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(\left(\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}\right)^{3/2}c^{5/2}\sqrt{x^2\,\left(b+c\,x^2\right)}\right)$$

## Problem 263: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{5/2}\,\left(A+B\,x^2\right)}{\left(b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$-\frac{\left(b\,B-A\,c\right)\,x^{3/2}}{b\,c\,\sqrt{b\,x^2+c\,x^4}}+\left(\left(b\,B+A\,c\right)\,x\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,b^{5/4}\,c^{5/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 132 leaves):

$$\left(\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\,\left(-b\,B+A\,c\right)\,x^{3/2}+\mathrm{i}\,\left(b\,B+A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\,x^2\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(b\,\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\,c\,\sqrt{x^2\,\left(b+c\,x^2\right)}\right)$$

## Problem 264: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}\,\left(A+B\,x^2\right)}{\left(b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 318 leaves, 7 steps):

$$-\frac{2 A \sqrt{x}}{b \sqrt{b x^2+c x^4}}+\frac{(b B-3 A c) x^{5/2}}{b^2 \sqrt{b x^2+c x^4}}-\frac{(b B-3 A c) x^{3/2} \left(b+c x^2\right)}{b^2 \sqrt{c} \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{b x^2+c x^4}}+\left((b B-3 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(b^{7/4} c^{3/4} \sqrt{b x^2+c x^4}\right)-\left((b B-3 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(2 b^{7/4} c^{3/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 203 leaves):

$$\left(i x^{3/2} \left(\sqrt{c} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \left(-2 A b+b B x^2-3 A c x^2\right)-\sqrt{b} (b B-3 A c) x \sqrt{1+\frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]+\sqrt{b} (b B-3 A c) x \sqrt{1+\frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]\right)\right) \Big/ \left(b^{5/2} \left(\frac{i \sqrt{c} x}{\sqrt{b}}\right)^{3/2} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

## Problem 265: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x} \left(A+B x^2\right)}{\left(b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$-\frac{2 A}{3 b \sqrt{x} \sqrt{b x^2+c x^4}}+\frac{(3 b B-5 A c) x^{3/2}}{3 b^2 \sqrt{b x^2+c x^4}}+\left((3 b B-5 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(6 b^{9/4} c^{1/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 147 leaves) :

$$\left(\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\left(-2\,A\,b+3\,b\,B\,x^2-5\,A\,c\,x^2\right)-\mathrm{i}\left(-3\,b\,B+5\,A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\;x^{5/2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Bigg/\left(3\,b^2\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{c}}}\;\sqrt{x}\;\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

## Problem 266: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{\sqrt{x}\,\left(b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 368 leaves, 8 steps) :

$$-\frac{2\,A}{5\,b\,x^{3/2}\sqrt{b\,x^2+c\,x^4}}+\frac{\left(5\,b\,B-7\,A\,c\right)\sqrt{x}}{5\,b^2\sqrt{b\,x^2+c\,x^4}}+\frac{3\sqrt{c}\,\left(5\,b\,B-7\,A\,c\right)x^{3/2}\left(b+c\,x^2\right)}{5\,b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{b\,x^2+c\,x^4}}-\frac{3\left(5\,b\,B-7\,A\,c\right)\sqrt{b\,x^2+c\,x^4}}{5\,b^3\,x^{3/2}}-\left(3\,c^{1/4}\left(5\,b\,B-7\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/\left(5\,b^{11/4}\sqrt{b\,x^2+c\,x^4}\right)+\left(3\,c^{1/4}\left(5\,b\,B-7\,A\,c\right)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/\left(10\,b^{11/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 236 leaves) :

$$\left(\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\left(-5\,b\,B\,x^2\left(2\,b+3\,c\,x^2\right)+A\left(-2\,b^2+14\,b\,c\,x^2+21\,c^2\,x^4\right)\right)+\right.$$
$$3\sqrt{b}\,\sqrt{c}\,\left(5\,b\,B-7\,A\,c\right)\,x^3\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]-$$
$$\left.3\sqrt{b}\,\sqrt{c}\,\left(5\,b\,B-7\,A\,c\right)\,x^3\sqrt{1+\frac{c\,x^2}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\right],-1\right]\right)\Bigg/$$
$$\left(5\,b^3\,x^{3/2}\sqrt{\frac{i\sqrt{c}\,x}{\sqrt{b}}}\,\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

## Problem 267: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^2}{x^{3/2}\left(b\,x^2+c\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 203 leaves, 6 steps):

$$-\frac{2\,A}{7\,b\,x^{5/2}\sqrt{b\,x^2+c\,x^4}}+\frac{7\,b\,B-9\,A\,c}{7\,b^2\sqrt{x}\,\sqrt{b\,x^2+c\,x^4}}-\frac{5\left(7\,b\,B-9\,A\,c\right)\sqrt{b\,x^2+c\,x^4}}{21\,b^3\,x^{5/2}}-$$
$$\left(5\,c^{3/4}\left(7\,b\,B-9\,A\,c\right)\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c\,x^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/$$
$$\left(42\,b^{13/4}\sqrt{b\,x^2+c\,x^4}\right)$$

Result (type 4, 170 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(-7\,b\,B\,x^2\left(2\,b+5\,c\,x^2\right)+A\left(-6\,b^2+18\,b\,c\,x^2+45\,c^2\,x^4\right)\right)+\right.$$
$$\left.5\,i\,c\left(-7\,b\,B+9\,A\,c\right)\sqrt{1+\frac{b}{c\,x^2}}\,x^{9/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Bigg/$$
$$\left(21\,b^3\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\,x^{5/2}\sqrt{x^2\left(b+c\,x^2\right)}\right)$$

## Problem 268: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{x^{5/2} \left(b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 405 leaves, 9 steps):

$$-\frac{2 A}{9 b x^{7/2} \sqrt{b x^2+c x^4}}+\frac{9 b B-11 A c}{9 b^2 x^{3/2} \sqrt{b x^2+c x^4}}-\frac{7 c^{3/2} (9 b B-11 A c) x^{3/2} \left(b+c x^2\right)}{15 b^4 \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{b x^2+c x^4}}-\frac{7 (9 b B-11 A c) \sqrt{b x^2+c x^4}}{45 b^3 x^{7/2}}+\frac{7 c (9 b B-11 A c) \sqrt{b x^2+c x^4}}{15 b^4 x^{3/2}}+\left(7 c^{5/4} (9 b B-11 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(15 b^{15/4} \sqrt{b x^2+c x^4}\right)-\left(7 c^{5/4} (9 b B-11 A c) x \left(\sqrt{b}+\sqrt{c} x\right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(30 b^{15/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 259 leaves):

$$\left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \left(9 b B x^2 \left(-2 b^2+14 b c x^2+21 c^2 x^4\right)-A \left(10 b^3-22 b^2 c x^2+154 b c^2 x^4+231 c^3 x^6\right)\right)-21 \sqrt{b} c^{3/2} (9 b B-11 A c) x^5 \sqrt{1+\frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]+21 \sqrt{b} c^{3/2} (9 b B-11 A c) x^5 \sqrt{1+\frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]\right) \Big/ \left(45 b^4 x^{7/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 \left(b+c x^2\right)}\right)$$

## Problem 276: Result more than twice size of optimal antiderivative.

$$\int (e x)^m \left(c+d x^n\right)^q \left(a x^j+b x^{j+n}\right)^p \, dx$$

Optimal (type 6, 113 leaves, 4 steps):

$$\frac{1}{1+m+j\,p}x\,(e\,x)^m\left(1+\frac{b\,x^n}{a}\right)^{-p}(c+d\,x^n)^q\left(1+\frac{d\,x^n}{c}\right)^{-q}$$
$$(a\,x^j+b\,x^{j+n})^p\,\text{AppellF1}\left[\frac{1+m+j\,p}{n},-p,-q,\frac{1+m+n+j\,p}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]$$

Result (type 6, 267 leaves):

$$\left(a\,c\,(1+m+n+j\,p)\,x\,(e\,x)^m\left(x^j\,(a+b\,x^n)\right)^p\right.$$
$$\left.(c+d\,x^n)^q\,\text{AppellF1}\left[\frac{1+m+j\,p}{n},-p,-q,\frac{1+m+n+j\,p}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\Big/$$
$$\left((1+m+j\,p)\left(a\,c\,(1+m+n+j\,p)\,\text{AppellF1}\left[\frac{1+m+j\,p}{n},-p,-q,\frac{1+m+n+j\,p}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+\right.\right.$$
$$n\,x^n\left(b\,c\,p\,\text{AppellF1}\left[\frac{1+m+n+j\,p}{n},1-p,-q,\frac{1+m+2\,n+j\,p}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+\right.$$
$$\left.\left.\left.a\,d\,q\,\text{AppellF1}\left[\frac{1+m+n+j\,p}{n},-p,1-q,\frac{1+m+2\,n+j\,p}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\right)\right)$$

## Problem 277: Result more than twice size of optimal antiderivative.

$$\int(e\,x)^{7/4}\,(c+d\,x^n)^q\,\left(a\,x^j+b\,x^{j+n}\right)^{5/3}\,dx$$

Optimal (type 6, 129 leaves, 4 steps):

$$\left(12\,a\,e\,x^{2+j}\,(e\,x)^{3/4}\,(c+d\,x^n)^q\left(1+\frac{d\,x^n}{c}\right)^{-q}\left(a\,x^j+b\,x^{j+n}\right)^{2/3}\right.$$
$$\left.\text{AppellF1}\left[\frac{33+20\,j}{12\,n},-\frac{5}{3},-q,\frac{33+20\,j+12\,n}{12\,n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\Big/\left((33+20\,j)\left(1+\frac{b\,x^n}{a}\right)^{2/3}\right)$$

Result (type 6, 580 leaves):

$$\frac{1}{33+20j+12n}12\,a\,c\,x^{1+j}\,(e\,x)^{7/4}\,\left(x^j\,(a+b\,x^n)\right)^{2/3}\,(c+d\,x^n)^q$$
$$\left(\left(a\,(33+20j+12n)^2\,\text{AppellF1}\left[\frac{33+20j}{12n},-\frac{2}{3},-q,\frac{\frac{11}{4}+\frac{5j}{3}+n}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\Big/\left((33+20j)\right.\right.$$
$$\left(a\,c\,(33+20j+12n)\,\text{AppellF1}\left[\frac{33+20j}{12n},-\frac{2}{3},-q,\frac{33+20j+12n}{12n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+\right.$$
$$4\,n\,x^n\left(3\,a\,d\,q\,\text{AppellF1}\left[\frac{33+20j+12n}{12n},-\frac{2}{3},1-q,\frac{33+20j+24n}{12n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+\right.$$
$$\left.\left.\left.2\,b\,c\,\text{AppellF1}\left[\frac{33+20j+12n}{12n},\frac{1}{3},-q,\frac{33+20j+24n}{12n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\right)\right)+$$
$$\left(b\,(33+20j+24n)\,x^n\,\text{AppellF1}\left[\frac{33+20j+12n}{12n},-\frac{2}{3},-q,\frac{33+20j+24n}{12n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\Big/$$
$$\left(a\,c\,(33+20j+24n)\,\text{AppellF1}\left[\frac{33+20j+12n}{12n},-\frac{2}{3},-q,\frac{33+20j+24n}{12n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+\right.$$
$$4\,n\,x^n\left(3\,a\,d\,q\,\text{AppellF1}\left[\frac{33+20j+24n}{12n},-\frac{2}{3},1-q,\frac{33+20j+36n}{12n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+\right.$$
$$\left.\left.\left.2\,b\,c\,\text{AppellF1}\left[\frac{33+20j+24n}{12n},\frac{1}{3},-q,\frac{33+20j+36n}{12n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\right)\right)$$

## Problem 282: Unable to integrate problem.

$$\int\frac{a\,x^m+b\,x^n}{c\,x^m+d\,x^n}\,dx$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{a\,x}{c}+\frac{(b\,c-a\,d)\,x\,\text{Hypergeometric2F1}\left[1,\frac{1}{m-n},1+\frac{1}{m-n},-\frac{c\,x^{m-n}}{d}\right]}{c\,d}$$

Result (type 8, 27 leaves):

$$\int\frac{a\,x^m+b\,x^n}{c\,x^m+d\,x^n}\,dx$$

## Problem 284: Unable to integrate problem.

$$\int\frac{\left(a+\frac{b}{x}\right)^n x^m}{c+d\,x}\,dx$$

Optimal (type 6, 64 leaves, 4 steps):

$$\frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{a\,x}\right)^{-n}x^m\,\text{AppellF1}\left[-m,-n,1,1-m,-\frac{b}{a\,x},-\frac{c}{d\,x}\right]}{d\,m}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+d\,x}\,dx$$

## Problem 285: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+d\,x}\,dx$$

Optimal (type 5, 195 leaves, 7 steps):

$$-\frac{\left(2\,a\,c+b\,d\,(1-n)\right)\left(a+\frac{b}{x}\right)^{1+n} x}{2\,a^2\,d^2}+\frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2\,a\,d}-\frac{c^3\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{c\left(a+\frac{b}{x}\right)}{a\,c-b\,d}\right]}{d^3\,(a\,c-b\,d)\,(1+n)}+\frac{1}{2\,a^3\,d^3\,(1+n)}\left(2\,a^2\,c^2-2\,a\,b\,c\,d\,n-b^2\,d^2\,(1-n)\,n\right)\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,1+\frac{b}{a\,x}\right]$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+d\,x}\,dx$$

## Problem 286: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+d\,x}\,dx$$

Optimal (type 5, 131 leaves, 6 steps):

$$\frac{\left(a+\frac{b}{x}\right)^{1+n} x}{a\,d}+\frac{c^2\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{c\left(a+\frac{b}{x}\right)}{a\,c-b\,d}\right]}{d^2\,(a\,c-b\,d)\,(1+n)}-\frac{(a\,c-b\,d\,n)\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,1+\frac{b}{a\,x}\right]}{a^2\,d^2\,(1+n)}$$

Result (type 8, 20 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+d\,x}\,dx$$

## Problem 287: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+d x} \, dx$$

Optimal (type 5, 101 leaves, 5 steps):

$$-\frac{c \left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{c \left(a+\frac{b}{x}\right)}{a c-b d}\right]}{d (a c-b d) (1+n)}+\frac{\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,1+\frac{b}{a x}\right]}{a d (1+n)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+d x} \, dx$$

## Problem 288: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x (c+d x)} \, dx$$

Optimal (type 5, 54 leaves, 3 steps):

$$\frac{\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{c \left(a+\frac{b}{x}\right)}{a c-b d}\right]}{(a c-b d) (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x (c+d x)} \, dx$$

## Problem 289: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2 (c+d x)} \, dx$$

Optimal (type 5, 84 leaves, 4 steps):

$$-\frac{\left(a+\frac{b}{x}\right)^{1+n}}{b c (1+n)}-\frac{d \left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{c \left(a+\frac{b}{x}\right)}{a c-b d}\right]}{c (a c-b d) (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+d\,x)}\,dx$$

## Problem 290: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(c+d\,x)}\,dx$$

Optimal (type 5, 115 leaves, 5 steps):

$$\frac{(a\,c+b\,d)\,\left(a+\frac{b}{x}\right)^{1+n}}{b^2\,c^2\,(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2\,c\,(2+n)}+\frac{d^2\,\left(a+\frac{b}{x}\right)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{c\,\left(a+\frac{b}{x}\right)}{a\,c-b\,d}\right]}{c^2\,(a\,c-b\,d)\,(1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(c+d\,x)}\,dx$$

## Problem 291: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\,(c+d\,x)}\,dx$$

Optimal (type 5, 207 leaves, 5 steps):

$$\frac{(a\,c+b\,d)\,\left(a^2\,c^2+b^2\,d^2\right)\,\left(a+\frac{b}{x}\right)^{1+n}}{b^4\,c^4\,(1+n)}-\frac{\left(3\,a^2\,c^2+2\,a\,b\,c\,d+b^2\,d^2\right)\,\left(a+\frac{b}{x}\right)^{2+n}}{b^4\,c^3\,(2+n)}+$$
$$\frac{(3\,a\,c+b\,d)\,\left(a+\frac{b}{x}\right)^{3+n}}{b^4\,c^2\,(3+n)}-\frac{\left(a+\frac{b}{x}\right)^{4+n}}{b^4\,c\,(4+n)}+\frac{d^4\,\left(a+\frac{b}{x}\right)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{c\,\left(a+\frac{b}{x}\right)}{a\,c-b\,d}\right]}{c^4\,(a\,c-b\,d)\,(1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\,(c+d\,x)}\,dx$$

## Problem 292: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n\,x^m}{(c+d\,x)^2}\,dx$$

Optimal (type 6, 73 leaves, 4 steps):

$$-\frac{\left(a+\frac{b}{x}\right)^n\,\left(1+\frac{b}{a\,x}\right)^{-n}\,x^{-1+m}\,\text{AppellF1}\left[1-m,\,-n,\,2,\,2-m,\,-\frac{b}{a\,x},\,-\frac{c}{d\,x}\right]}{d^2\,(1-m)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+d x)^2} \, dx$$

## Problem 293: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+d x)^2} \, dx$$

Optimal (type 5, 202 leaves, 7 steps):

$$\frac{c\,(2 a c-b d)\left(a+\frac{b}{x}\right)^{1+n}}{a d^2 (a c-b d)\left(d+\frac{c}{x}\right)}+\frac{\left(a+\frac{b}{x}\right)^{1+n} x}{a d\left(d+\frac{c}{x}\right)}+$$
$$\left(c^2 (2 a c-b d (2-n))\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{a c-b d}\right]\right)\Big/$$
$$\left(d^3 (a c-b d)^2 (1+n)\right)-\frac{(2 a c-b d n)\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,1+\frac{b}{a x}\right]}{a^2 d^3 (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+d x)^2} \, dx$$

## Problem 294: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+d x)^2} \, dx$$

Optimal (type 5, 150 leaves, 6 steps):

$$-\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d (a c-b d)\left(d+\frac{c}{x}\right)}-$$
$$\left(c\,(a c-b d (1-n))\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{a c-b d}\right]\right)\Big/$$
$$\left(d^2 (a c-b d)^2 (1+n)\right)+\frac{\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,1+\frac{b}{a x}\right]}{a d^2 (1+n)}$$

Result (type 8, 20 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+d x)^2} \, dx$$

## Problem 295: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+d x)^2} \, dx$$

Optimal (type 5, 56 leaves, 3 steps):

$$-\frac{b \left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{c \left(a+\frac{b}{x}\right)}{a c-b d}\right]}{(a c-b d)^2 (1+n)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+d x)^2} \, dx$$

## Problem 296: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x (c+d x)^2} \, dx$$

Optimal (type 5, 105 leaves, 4 steps):

$$-\frac{d \left(a+\frac{b}{x}\right)^{1+n}}{c (a c-b d) \left(d+\frac{c}{x}\right)}+$$
$$\left((a c-b d (1+n)) \left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a+\frac{b}{x}\right)}{a c-b d}\right]\right) \Bigg/$$
$$\left(c (a c-b d)^2 (1+n)\right)$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x (c+d x)^2} \, dx$$

## Problem 297: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2 (c+d x)^2} \, dx$$

Optimal (type 5, 133 leaves, 5 steps):

$$-\frac{\left(a+\frac{b}{x}\right)^{1+n}}{b\,c^2\,(1+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2\,(a\,c-b\,d)\left(d+\frac{c}{x}\right)}-\left(d\,(2\,a\,c-b\,d\,(2+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{a\,c-b\,d}\right]\right)\Big/\left(c^2\,(a\,c-b\,d)^2\,(1+n)\right)$$

Result (type 8, 22 leaves):

$$\int\frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+d\,x)^2}\,dx$$

## Problem 298: Unable to integrate problem.

$$\int\frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(c+d\,x)^2}\,dx$$

Optimal (type 5, 217 leaves, 5 steps):

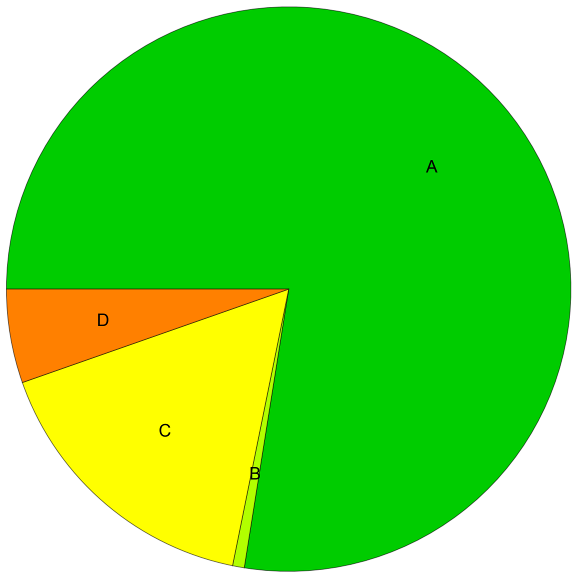
$$-\left(\left(\left(a+\frac{b}{x}\right)^{1+n}\left(d\,(b\,d\,(2+n)\,(a\,c+b\,d\,(3+n))-a\,c\,(a\,c+b\,d\,(5+3\,n)))-\frac{c\,(a\,c-b\,d)\,(a\,c+b\,d\,(3+n))}{x}\right)\right)\Big/\left(b^2\,c^3\,(a\,c-b\,d)\,(1+n)\,(2+n)\left(d+\frac{c}{x}\right)\right)\right)-\frac{\left(a+\frac{b}{x}\right)^{1+n}}{b\,c\,(2+n)\left(d+\frac{c}{x}\right)x^2}+\left(d^2\,(3\,a\,c-b\,d\,(3+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{a\,c-b\,d}\right]\right)\Big/\left(c^3\,(a\,c-b\,d)^2\,(1+n)\right)$$

Result (type 8, 22 leaves):

$$\int\frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(c+d\,x)^2}\,dx$$

# Summary of Integration Test Results

298 integration problems




A - 231 optimal antiderivatives

B - 2 more than twice size of optimal antiderivatives

C - 49 unnecessarily complex antiderivatives

D - 16 unable to integrate problems

E - 0 integration timeouts